Lecture 3

Topic: Mathematical and statistical model

Purpose of the lecture:

a) Students will have to understand the necessity of a modeling technology in social real project work.
b) To understand the concept of Mathematical and statistical model

Basic terms of the lecture:

Modeling; Mathematical models; Statistical model; Differential equation

Short abstracts:

Now a days modeling technology is developed very fast, and applied in a variety of aspects. What is model? How to model, and the principles of model is the most important concepts of the modeling technology. Mathematical and statistical model is the most important step of modeling technology, so it will help us to improve our speed of problems. In this lecture, we must to know about how to design a mathematical and statistical model for given task.

Main questions of the lecture:

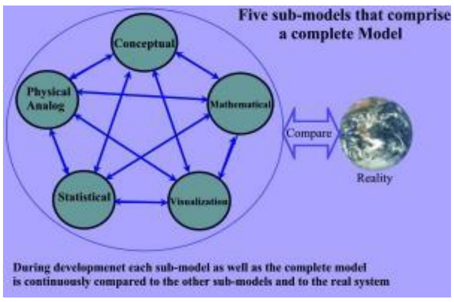
1) Why we study Mathematical and statistical model technology.
2) Describe the main purpose of the Mathematical and statistical model technology.
3) What is Mathematical and statistical model technology?
4) Describe the benefits of Mathematical and statistical model

Recommended list of literature sources:

1. Zeigler, B. P., T. G. Kim, and H. Praehofer. (2000). Theory of Modeling and Simulation. New York, NY, Academic Press.
2. Object Management Group, "Unified Modeling Language: Superstructure", Version 2.1.2, November 3, 2007.
3. Dubois, G. (2018) "Modeling and Simulation", Taylor & Francis, CRC Press.
4. Embley, D.W., Thalheim, B. (eds.): Handbook of Conceptual Modeling Theory, Practice, and Research Challenges. Springer, Berlin (2011)

Main contents of the lecture:

A model can come in many shapes, sizes, and styles. It is important to emphasize that a model is not the real world but merely a human construct to help us better understand real world systems. In general all models have an information input, an information processor, and an output of expected results. Modeling Methodology for Physics Teachers (more info) (1998) provides an outline of generic model structure that is useful for geoscience instruction. In "Modeling the Environment"



Andrew Ford gives a philosophical discussion of what models are and why they are useful. The first few paragraphs of Chapter 1 of Ford's book are worth a look.

Key features in common with the development of any model is that:

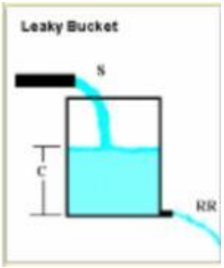
- simplifying assumptions must be made;
- boundary conditions or initial conditions must be identified;
- the range of applicability of the model should be understood.

Model Types:

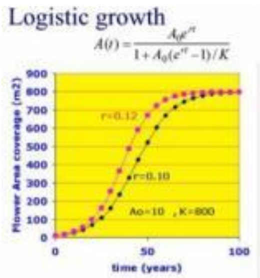
Below we identify 4 types of models for discussion and reference. Follow the link to a model type for an introduction to its use in the classroom and example activities. In practice **a well developed model of a real-world system will likely contain aspects of each individual model type described here.**

Conceptual Models are qualitative models that help highlight important connections in real world systems and processes. They are used as a first step in the development of more complex models.

Interactive Lecture Demonstrations Interactive demonstrations are physical models of systems that can be easily observed and manipulated and which have characteristics similar to key features of more complex systems in the real world. These models can help bridge the gap between conceptual models and models of more complex real world systems.



Mathematical and Statistical Models involve solving relevant equation(s) of a system or characterizing a system based upon its statisical parameters such as mean, mode, variance or regression coefficients. Mathematical models include Analytical models and Numerical Models. Statistical models are useful in helping identify patterns and underlying relationships between data sets.



Teaching with Visualizations By this we mean anything that can help one visualize how a system works. A visualization model can be a direct link between data and some graphic or image output or can be linked in series with some other type of model so to convert its output into a visually useful format. Examples include 1-, 2-, and 3-D graphics packages, map overlays, animations, image manipulation and image analysis.

Several additional quotes relevant to using models and developing theories include:

- "All models are wrong but some are useful." George E.P. Box

- "Make your theory as simple as possible, but no simpler." A. Einstein
- "For every complex question there is a simple and wrong solution." A. Einstein.

A **model** is a representation of structure in a physical system and/or its properties. It describes (or specifies) four types of structure, each with internal and external components:

- **systemic structure** specifies
  - composition (internal parts of the system)
  - environment (external agents linked to the system)
  - connections (external and internal causal links)
- **geometric structure** specifies
  - position with respect to a reference frame (external geometry)
  - configuration (geometric relations among the parts)
- **temporal structure** specifies change in state variables (system properties)
  - descriptive models represent change by explicit functions of time
  - causal models specify change by differential equations with interaction laws
- **interaction structure** specifies interaction laws expressing interactions among causal links, usually as function of state variables

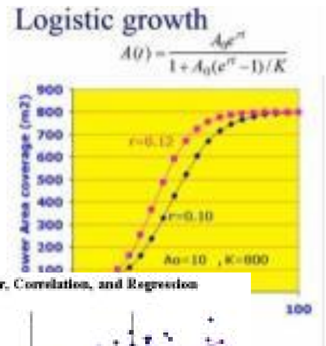
Mathematical and statistics model

What are Mathematical and Statistical Models

These types of models are obviously related, but there are also real differences between them.

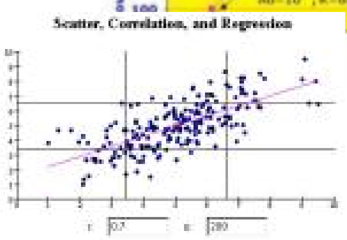
Mathematical Models:

Grow out of equations that determine how a system changes from one state to the next (**differential equations**) and/or how one variable depends on the value or state of other variables (**state equations**) These can also be divided into either numerical models or analytical models.



Statistical Models:

include issues such as statistical characterization of numerical data, estimating the probabilistic future behavior of a system based on past behavior, extrapolation or interpolation of data based on some best-fit, error estimates of observations, or spectral analysis of data or model generated output.



A Simple Mathematical Model

Here we use a simple example from introductory chemistry or physics as it should be familiar to many science educators. Assume that a gas is heated in a sealed flexible container so the pressure remains constant ($P=P_o$) and you want to develop a mathematical model describing how the volume of the container changes with time.

First, **Assumptions** are an essential part of all model development

- Heater power is constant and is inside the container so the gas begins heating up immediately.
- The gas mixes rapidly so that it heats uniformly
- The gas is an ideal diatomic gas (like air) so the molar heat capacity at constant pressure is (C_p=7/2R) & (PV=nRT is the **equation of state**)
- The gas **initially** has a pressure, temperature, and volume (P_o,T_o, V_o) of 1.0 atmosphere (1.013x10^5 Pa), 300K, and 1.0 liters (0.001 m^3). (initial conditions or boundary conditions must alway be considered).

The controlling **differential equation** of this system is:
Power = rate that Heat Energy is added = $nC_p(dT/dt)$

or $(dT/dt)=Power/(nC_p)$

This has the solution T = 300K + Power *time/(nC_p)

using the equation of state, $n=P_oV_o/(RT_o)$, and C_p=7/2R gives,

$T = T_o[1 + 2Power*time/(7P_oV_o)]$.

Finally using the equation of state again $V=nRT/P_o$

$V=V_o[1 +2Power*time/(7PoVo)]$

Units are also very important in mathematical models. In this example Power should be in Watts, time in seconds, Pressure in Pascals(Pa), temperature Kelvin, and Volume in cubic meters. Although in the final result the first V_o can be in liters while the V_o inside the square brackets must be 0.001 m^3.

Why use mathematical and statistical models to teach introductory courses?

Mathematical and Statistical models can be used to help students obtain a better grasp on a variety of topics.

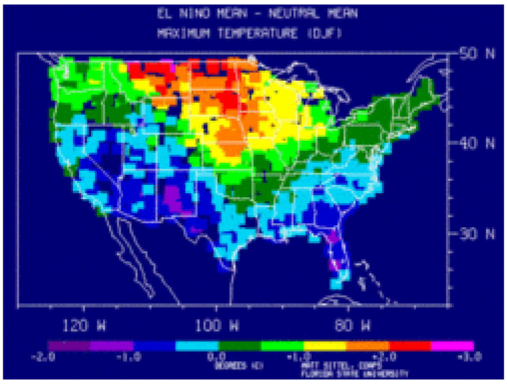
Mathematical Models

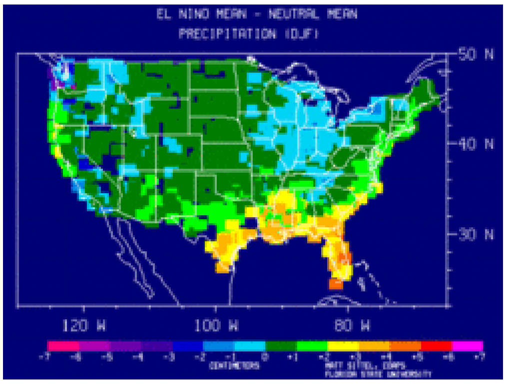
There are several situations in which mathematical models can be used very effectively in introductory education.

- Mathematical models can help students understand and explore the meaning of equations or functional relationships.
- Mathematical modeling software such as Excel, Stella II , or on-line JAVA /Macromedia type programs make it relatively easy to create a learning environment in which introductory students can be interactively engaged in guided inquiry, heads-on and hands-on activities.
- After developing a conceptual model of a physical system it is natural to develop a mathematical model that will allow one to estimate the quantitative behavior of the system.
- Quantitative results from mathematical models can easily be compared with observational data to identify a model's strengths and weaknesses.
- Mathematical models are an important component of the final "complete model" of a system which is actually a collection of conceptual, physical, mathematical, visualization, and possibly statistical sub-models.

Statistical Models

A solid statistical background is very important in the sciences. But the extent to which statistical ideas are appropriate in an introductory course depends on specific course objectives and the degree or institutional structure. Here we list several examples showing why and when statistical models are useful.





Statistical models or basic statistics can be used:

- To characterize numerical data to help one to concisely describe the measurements and to help in the development of conceptual models of a system or process;
- To help estimate uncertainties in observational data and uncertainties in calculation based on observational data;
- To characterize numerical output from mathematical models to help understand the model behavior and to assess the model's ability to simulate important features of the natural system(model validation). Feeding this information back into the model development process will enhance model performance;
- To estimate probabilistic future behavior of a system based on past statistical information, a **statistical prediction model**. This is often a method use in climate prediction. A statement like 'Southern California will be wet this winter because of a strong El Nino' is based on a statistical prediction model.
- To extrapolation or interpolation of data based on a linear fit (or some other mathematical fit) are also good examples of statistical prediction models.
- To estimate input parameters for more complex mathematical models.
- To obtain frequency spectra of observations and model output.

How to Use Mathematical and Statistical Models

General pedagogical considerations about using any type of model.

Technical considerations related to using different modeling environments specifically with mathematical and statistical models are outlined in each link below. These include tutorials on using the software, hardware considerations, and links to specific geoscience examples that use the respective modeling environments.

- Spreadsheets with a focus on Excel.
- Online interactive models based on either Java or Macromedia/Flash technology
- Stella II modeling environment.

How to Use Models

In thinking about how to incorporate modeling activities into introductory geoscience courses, there are two important classes of considerations: **technical** and **pedagogical.**

Technical Considerations include:

- Acquiring the models or ideas in a useable form.
- Identification and use of the proper equipment for physical demonstration models.

- In the case of mathematical models, computers simulations of analogous systems, visualization models, or statistical models one must learn how to operate and manipulate the modeling environment or software.

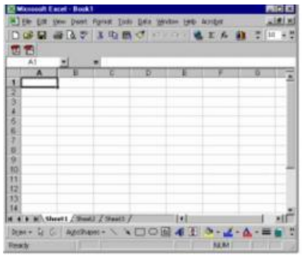
These technical considerations are clarified under our discussions related to each specific type of model.

Pedagogical Considerations

There are several things to keep in mind when using or creating modeling activities for instruction.

- Keep the activity as interactive as possible. When you find that you're spending a majority of your time lecturing to the students about what to do or how things work, try to think of ways you can get them working through ideas in groups, lab, interactive lectures, etc.
- Including students in the development process and/or providing opportunities for them to experiment with the model or modify it can increase students' understanding of the model and its relationship to the physical world.
- Creating opportunities for students to analyze and comment on the models behavior increases their understanding of the relationships between different inputs and rates.
- Creating opportunities for students to validate the model, i.e. compare model predictions to observations, increases their understanding of its limits.
- Stress that models are not reality and that a model's purpose is to help bridge the gap between observations and the real world. An important reason to use a model is that you can perform experiments with models without harming the system of interest.
- Make sure that students think about the underlying assumptions of a model and the domain of applicability. Try to ask questions that can help check their understanding. For example, simple exponential growth assumes that the percent growth rate remains fixed and in real world systems it only applies for so long before the system becomes overstressed. Having students identify underlying assumptions of a model and their domain of applicability can help them gain an appreciation of what a model can and cannot do.
- Models can be used to introduce specific content. A model can introduce students to important terms as well as provide an environment to explore relevant processes.
- Models can be used to explore "What-if" scenarios. "What if Atmospheric CO_2 doubles?" is a common example for a climate model.
- Models can be used explore the sensitivity of a system to variations in its different components.

What is Excel



Microsoft Excel is a spreadsheet program which allows one to enter numerical values or data into the rows or columns of a spreadsheet, and to use these numerical entries for such things as calculations, graphs, and statistical analysis.

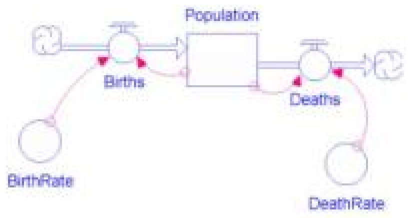
What are Interactive Online Models

Interactive online modeling environments are online web pages with an interactive graphical user interface for model input and control, dynamic graphs and tables for output, and animations. They are created to allow the user (student) control of a model or animation so they can perform virtual experiments. Interactive online modeling environments are typically written in JAVA or Macromedia Director programming languages. Most interactive online modeling environments are created to be platform and browser independent. However, there are still limited problems with some browsers/platform combinations. Although Java-Script (this is not related to JAVA) does not have animation capabilities it can be used to create interactive activities with text box input and output. Many interactive online calculators that can help students explore the behavior of equations and interactive online quizzes are created with Java-Script.

What is Stella II

The Stella II modeling Environment

Stella II software is an object-oriented programming environment. Models can be created with Stella II by connecting four or five different icons together in different ways into a model framework so that the structure of the model is very transparent. Shown at left is a simple population model with a limited carrying capacity. The model is constructed from Stocks (Population), flows (Births and Deaths), modifiers (BirthRate and DeathRate), and connectors. The software automatically creates the difference equations based on user input.



The mathematical details of the numerical solution to the underlying model equations are accessible for those that want this detail. Additional control over the time step used and the numerical solution technique is provided for the user.

```
Population(t) = Population(t - dt) + (Births - Deaths) * dt
INIT Population = .5

INFLOWS:
Births = BirthRate*Population
OUTFLOWS:
Deaths = Population*DeathRate*(1.0+Population/12.0)
BirthRate = 0.02
DeathRate = .01
```

The Stella II environment makes it easy for introductory geoscience students to visualize the conceptual framework of a model, manipulate the model to explore its dynamic behavior, and with a little experience create their own models. At left the graph of population over time is shown with a slider/text box input control for BirthRate.